AMSI VACATIONRESEARCH SCHOLARSHIPS 2020–21

Get a Thirst for Research this Summer

Identifying Lagrangian Coherent Structures with Fuzzy Consensus Clustering

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Vacation Research Scholarships are funded jointly by the Department of Education and Training and the Australian Mathematical Sciences Institute.

Australian Government

AMSI CONNECT

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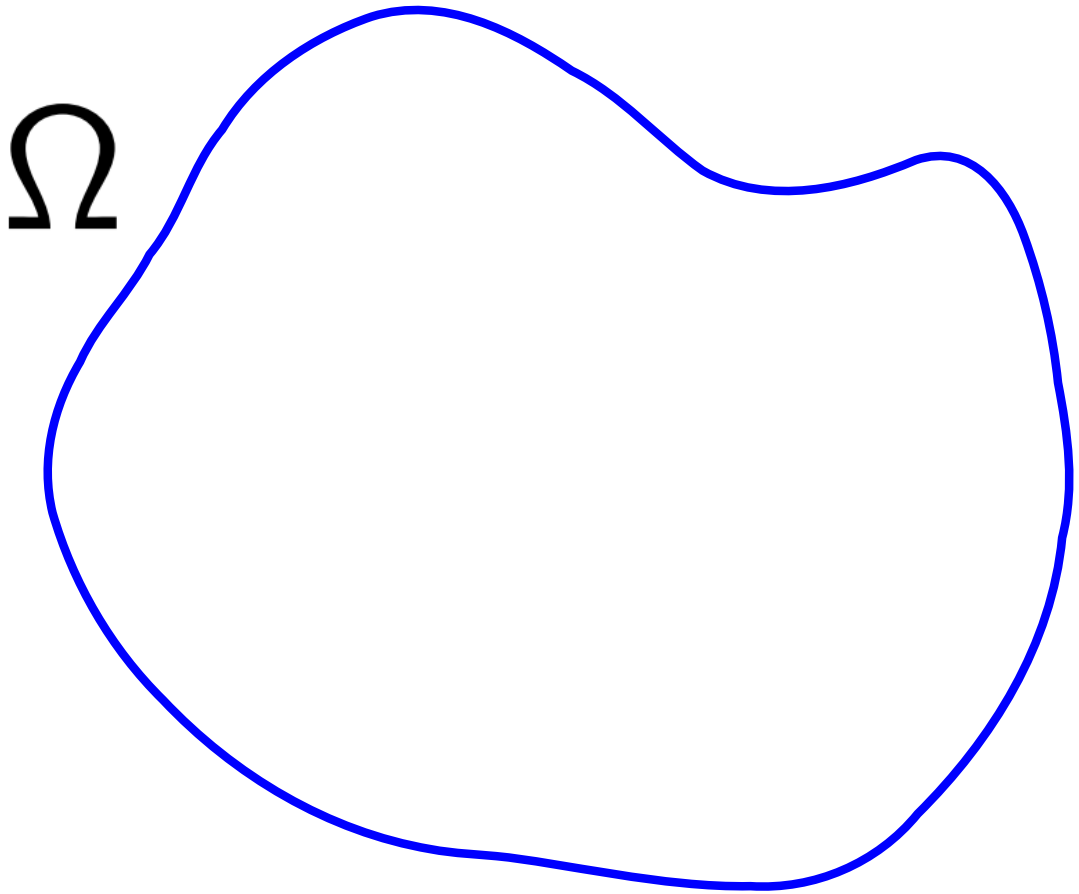
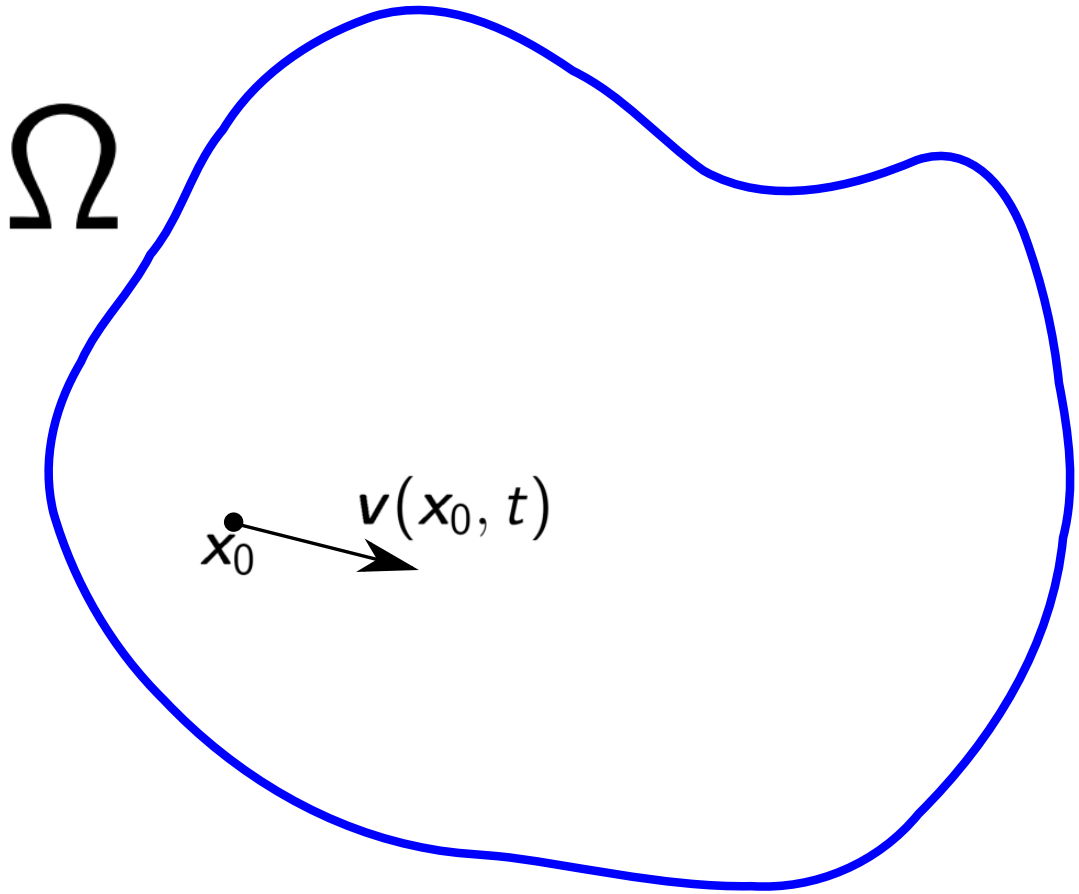
The Double Gyre

ANIMATION OF PARTICLES MOVING

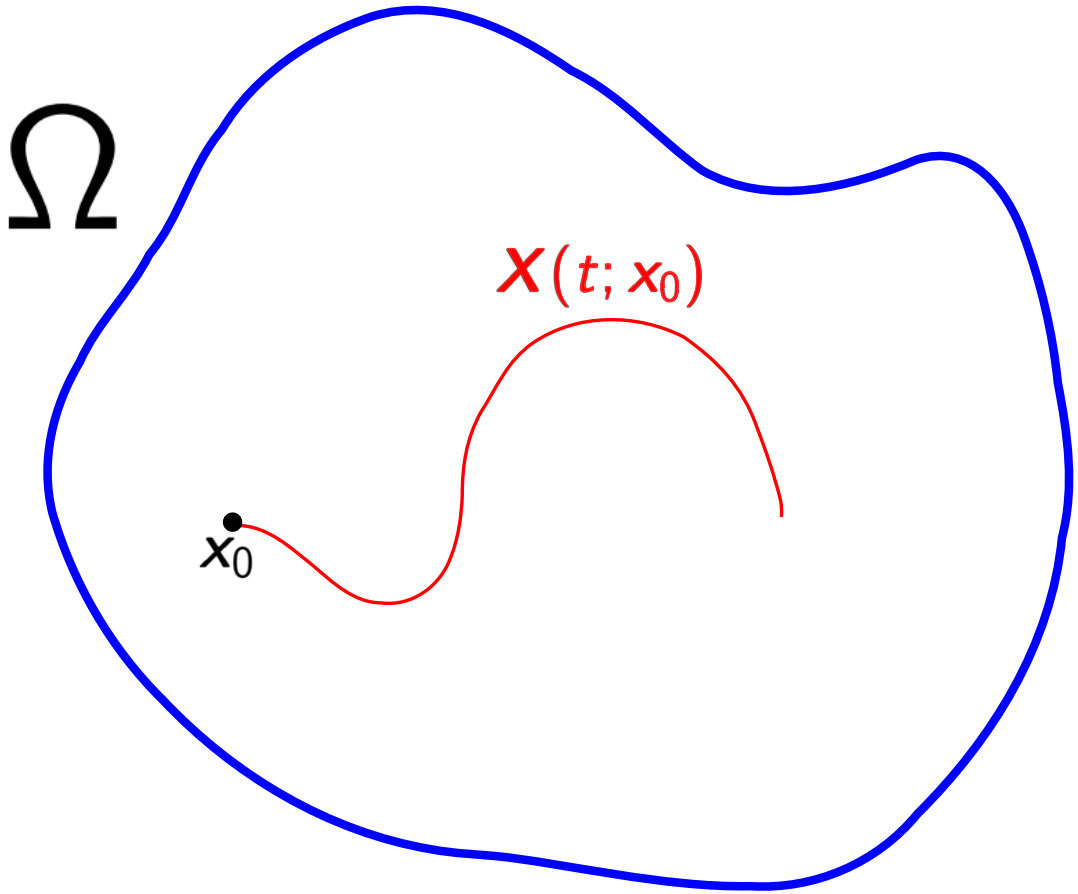
Mathematical Formulation



Mathematical Formulation



Mathematical Formulation



Lagrangian Coherent Structures (LCSs)

- **Lagrangian** - patterns in particle trajectories.

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- Wide range of diagnostic fields and techniques for extracting LCSs.

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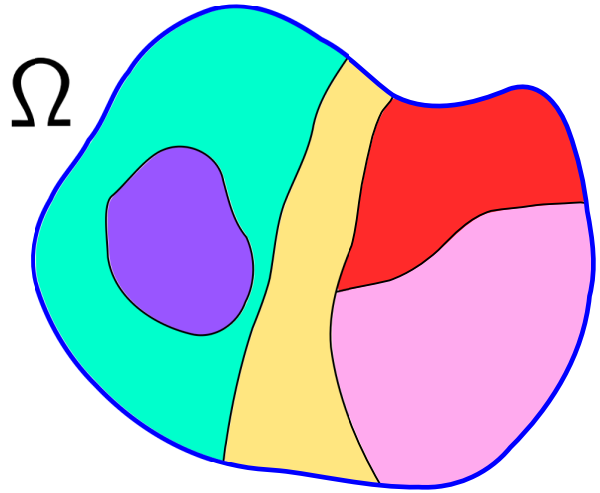
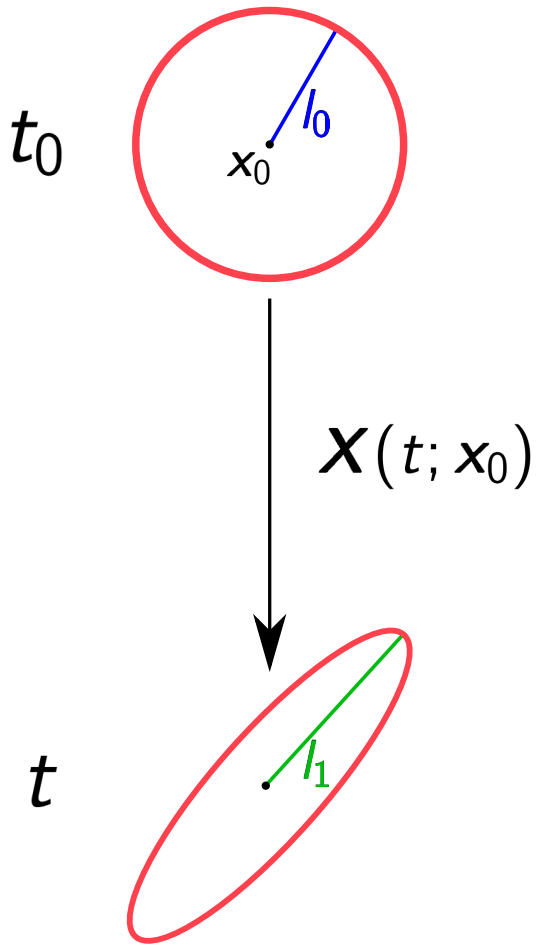
- **Lagrangian** - patterns in particle trajectories.
- Wide range of diagnostic fields and techniques for extracting LCSs.
- Some look to partition Ω into coherent regions.



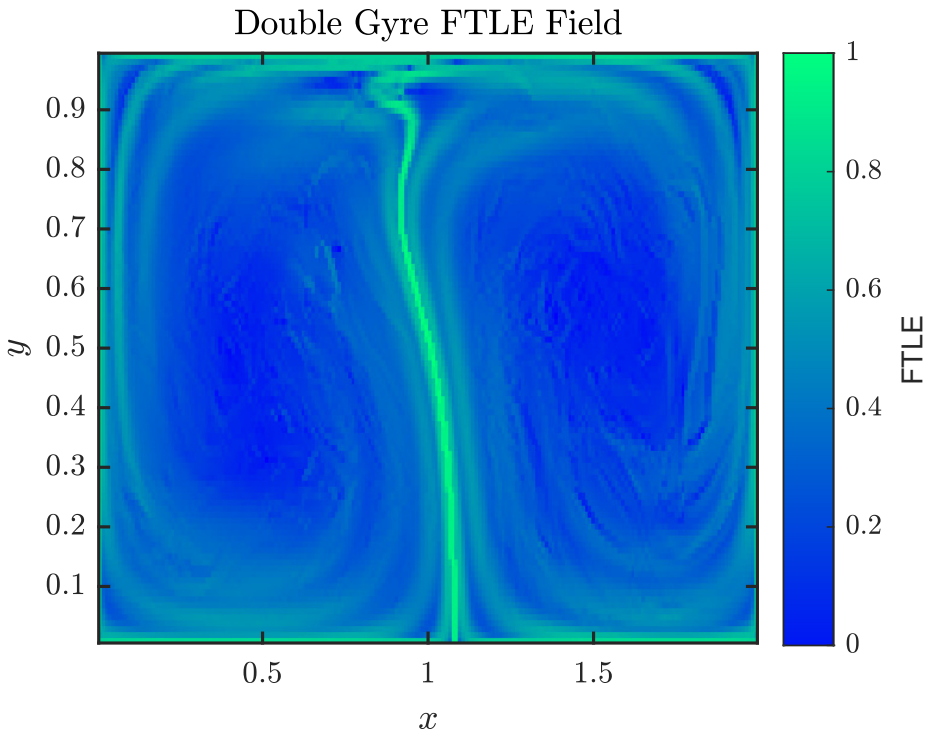
Finite-Time Lyapunov Exponent (FTLE)

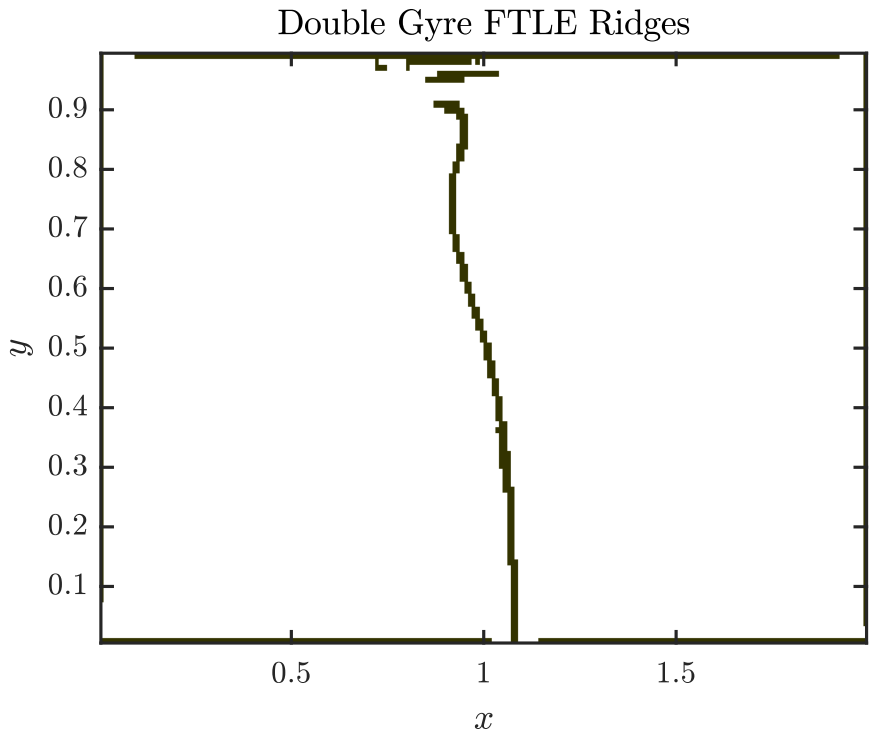


- Measure of maximal stretching of fluid.

$$\sigma := \frac{l_1}{l_0}$$

- One of the most common methods, established by Shadden et al. (2005).


Double Gyre FTLE Field
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
y
0.5
1
1.5
x
1
0.8
0.6
0.4
0.2
0
FTLE


Double Gyre FTLE Ridges
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
y
0.5
1
1.5
x

Lagrangian-Averaged Vorticity Deviation (LAVD)

- Vorticity measures local rotational rate.

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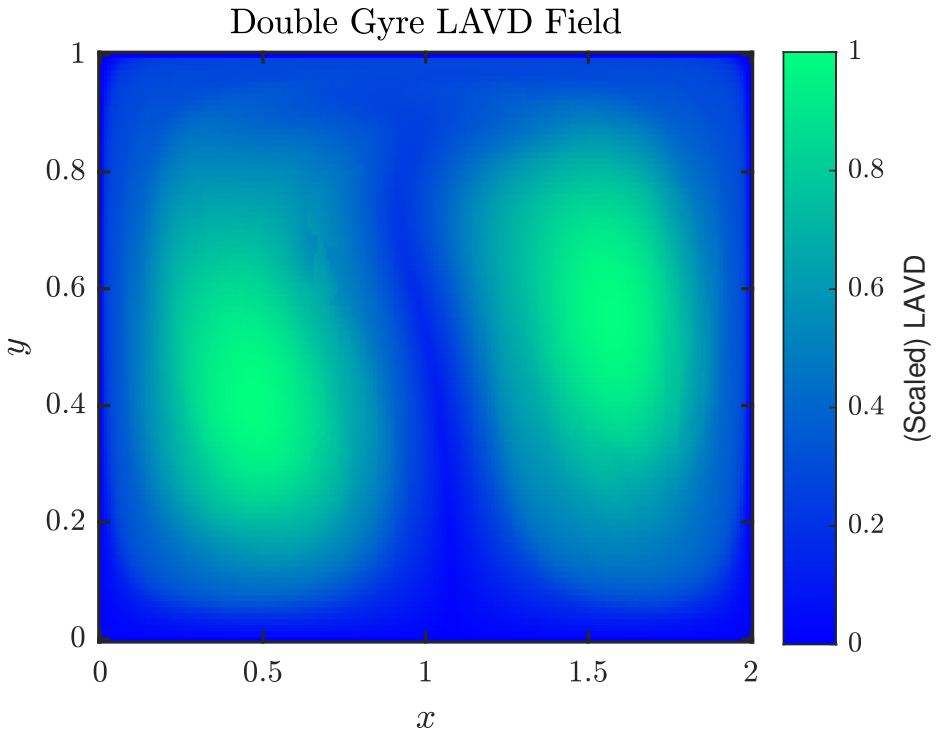
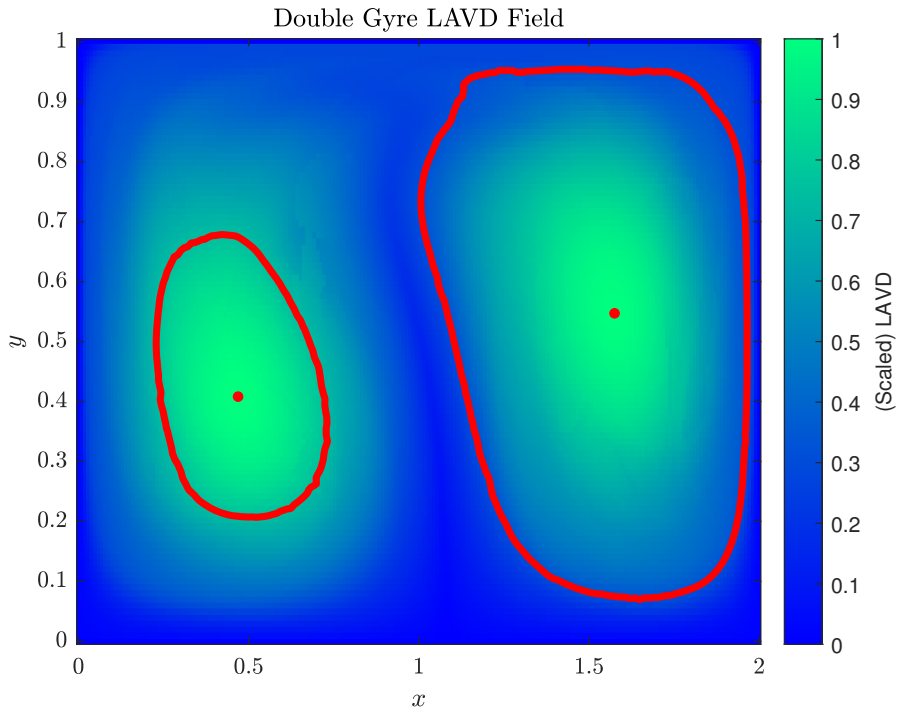
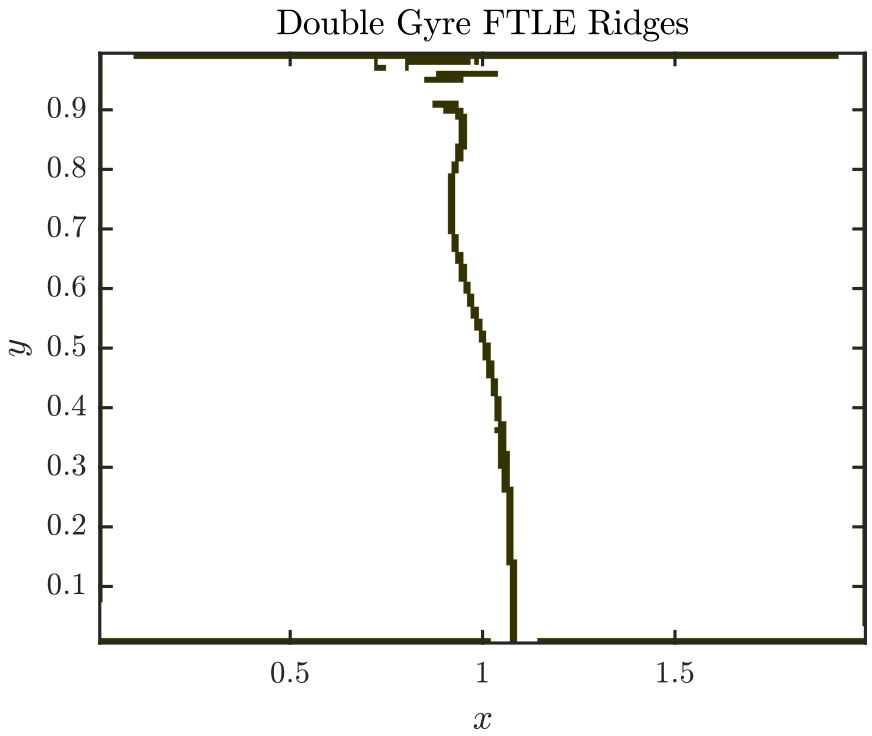
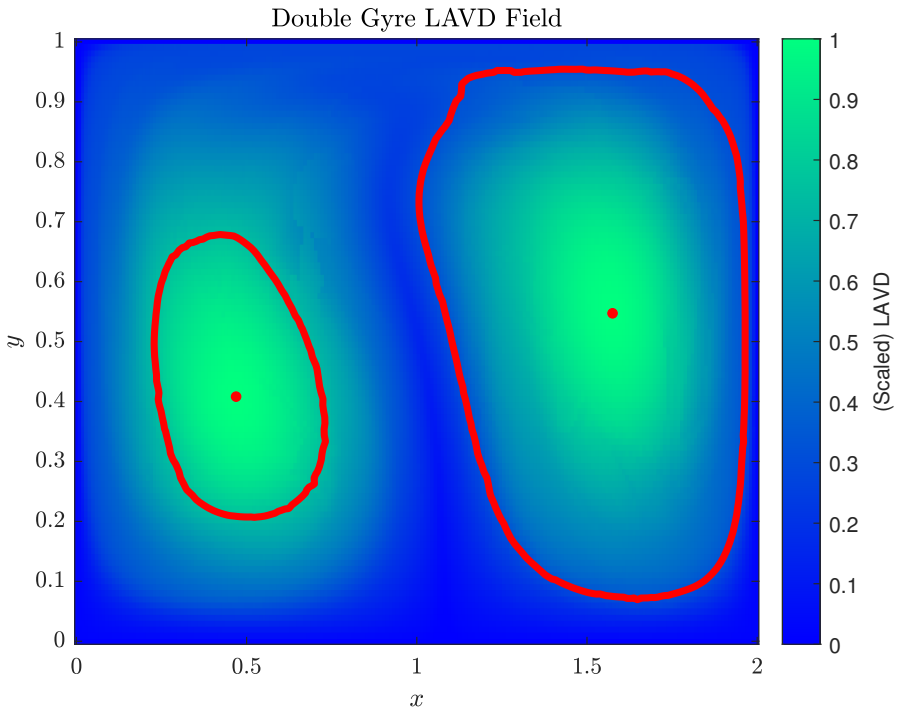
- Vorticity measures local rotational rate.
- LAVD is the vorticity relative to spatial mean, averaged along trajectory, proposed by Haller et al. (2016),


Double Gyre LAVD Field
y
x
(Scaled) LAVD
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2


Double Gyre LAVD Field
y
x
(Scaled) LAVD
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2


Double Gyre FTLE Ridges
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
y
0.5
1
1.5
x


Double Gyre LAVD Field
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.5
1
1.5
2
x
(Scaled) LAVD

Questions

Can we combine these perspectives into one way of looking at the flow?

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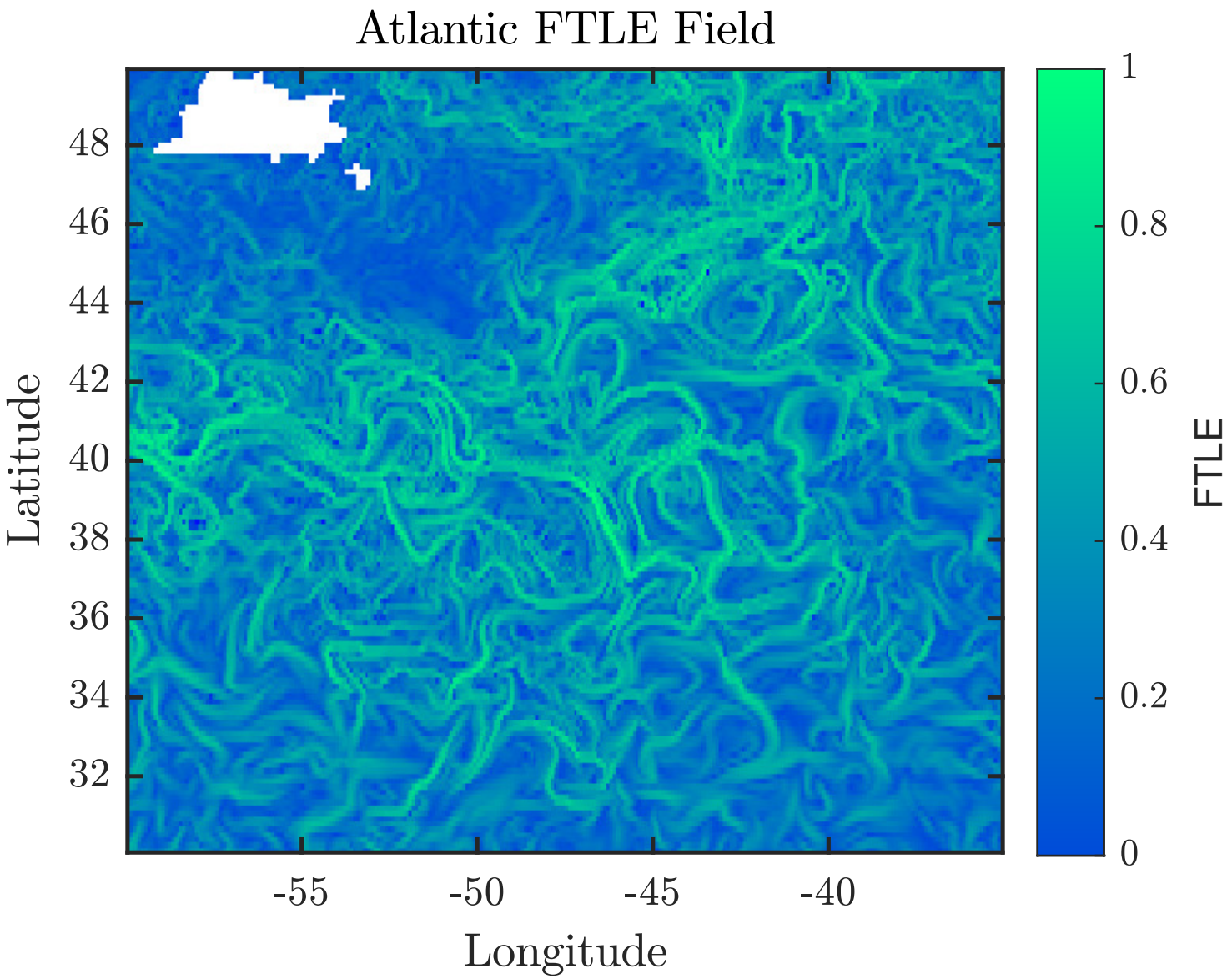
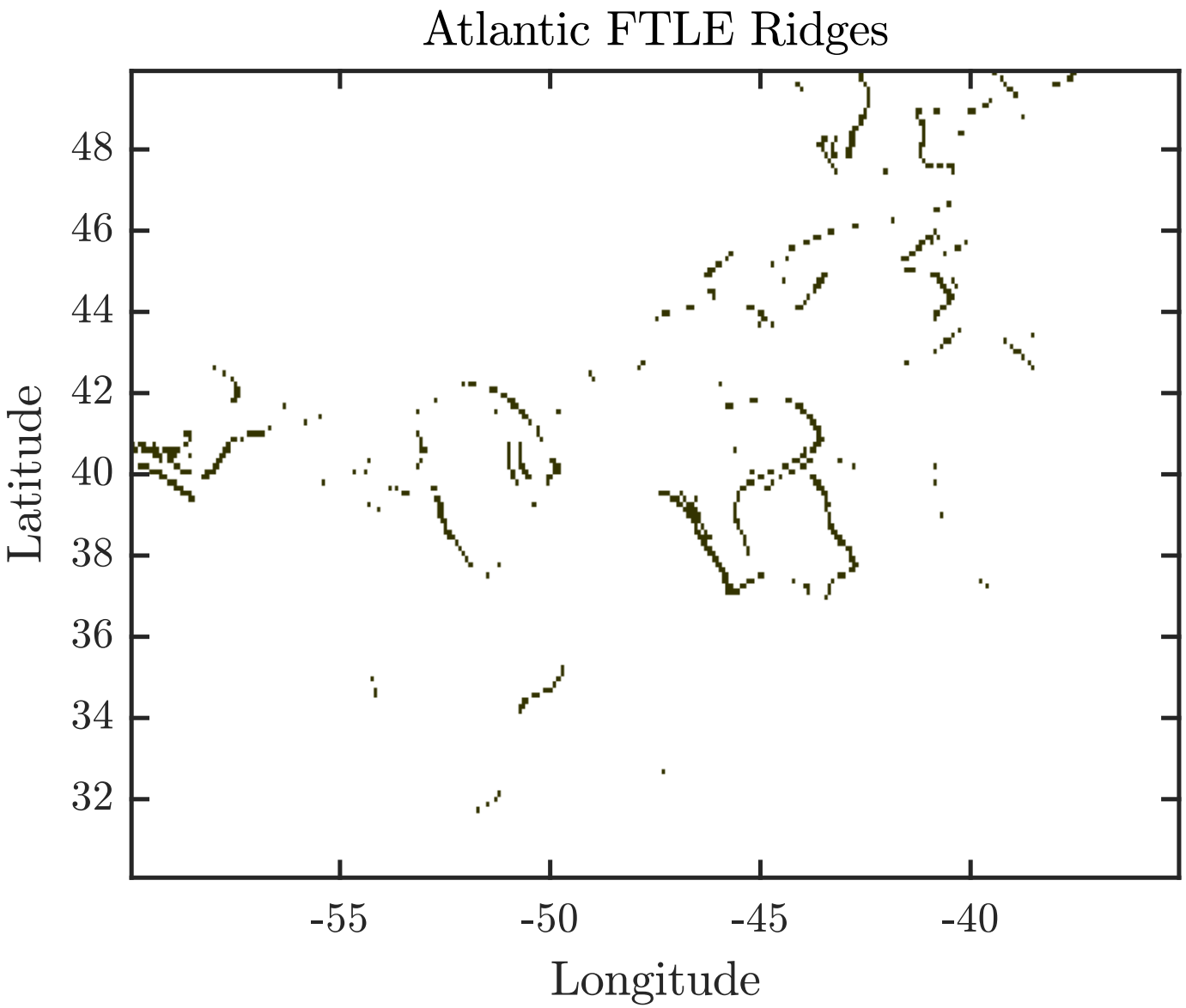
Can we combine these perspectives into one way of looking at the flow?

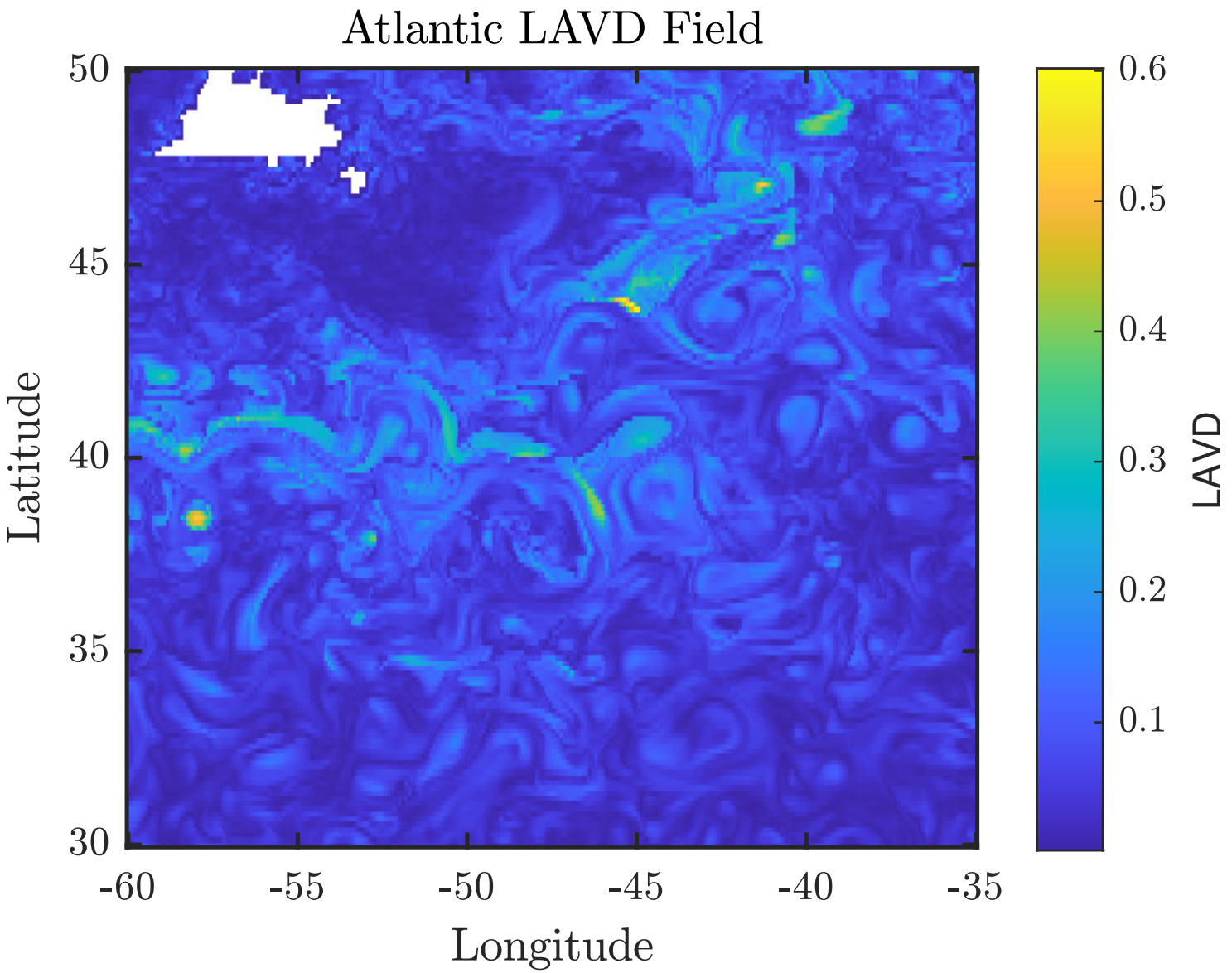
How does this look on real data?

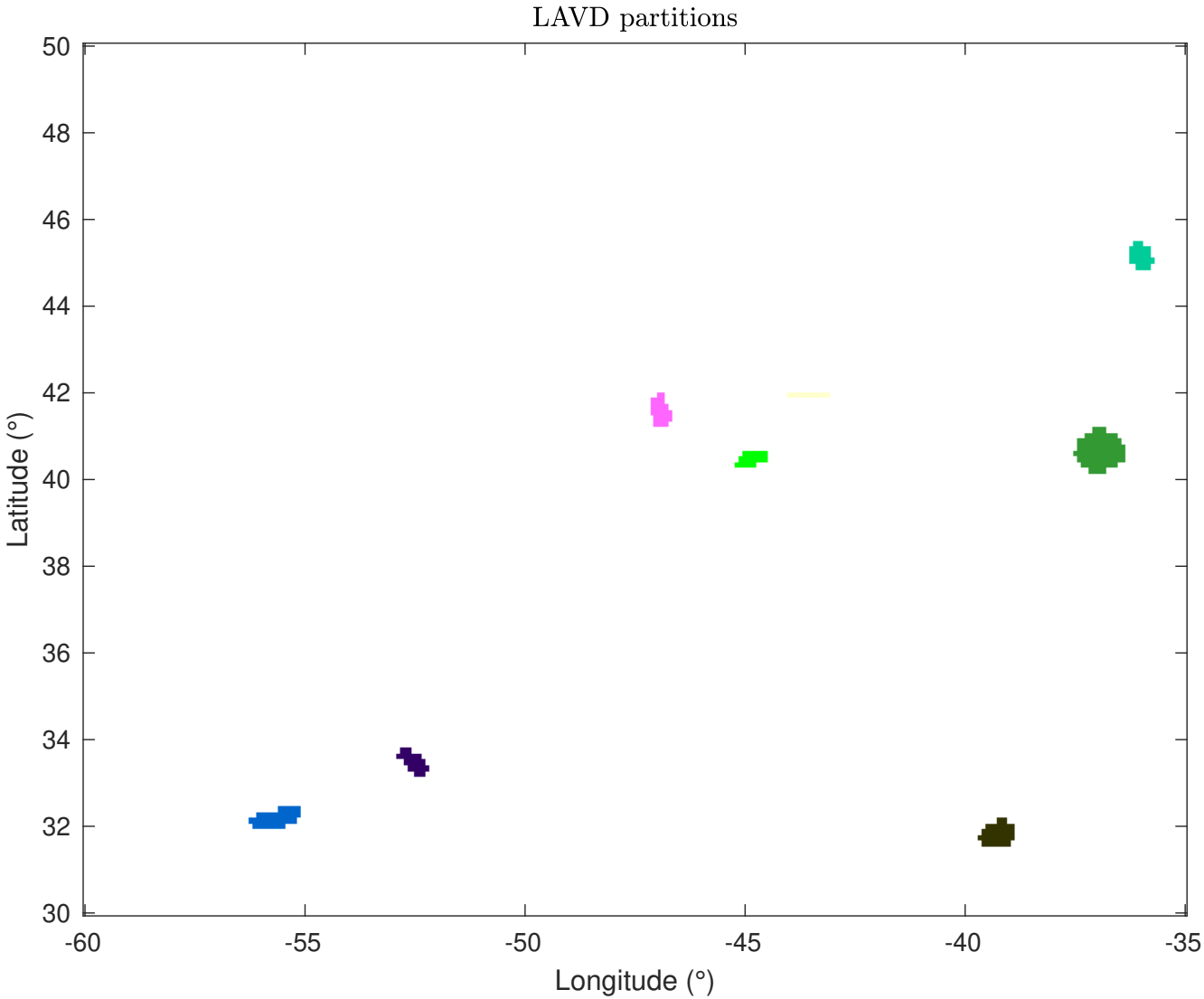
The Real Data

ANIMATION OF PARTICLES MOVING


Atlantic FTLE Field
Latitude
48
46
44
42
40
38
36
34
32
-55
-50
-45
-40
Longitude
1
0.8
0.6
0.4
0.2
0
FTLE


Atlantic FTLE Ridges
Latitude
48
46
44
42
40
38
36
34
32
-55
-50
-45
-40
Longitude


Atlantic LAVD Field
Latitude
50
45
40
35
30
-60
-55
-50
-45
-40
-35
Longitude
LAVD
0.6
0.5
0.4
0.3
0.2
0.1


LAVD partitions
Latitude (°)
Longitude (°)
50
48
46
44
42
40
38
36
34
32
30
-60
-55
-50
-45
-40
-35

How Can We Combine Perspectives: Clustering

- Grouping similar points together into clusters.

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- Key idea: **any** partitioning of the flow is a clustering.

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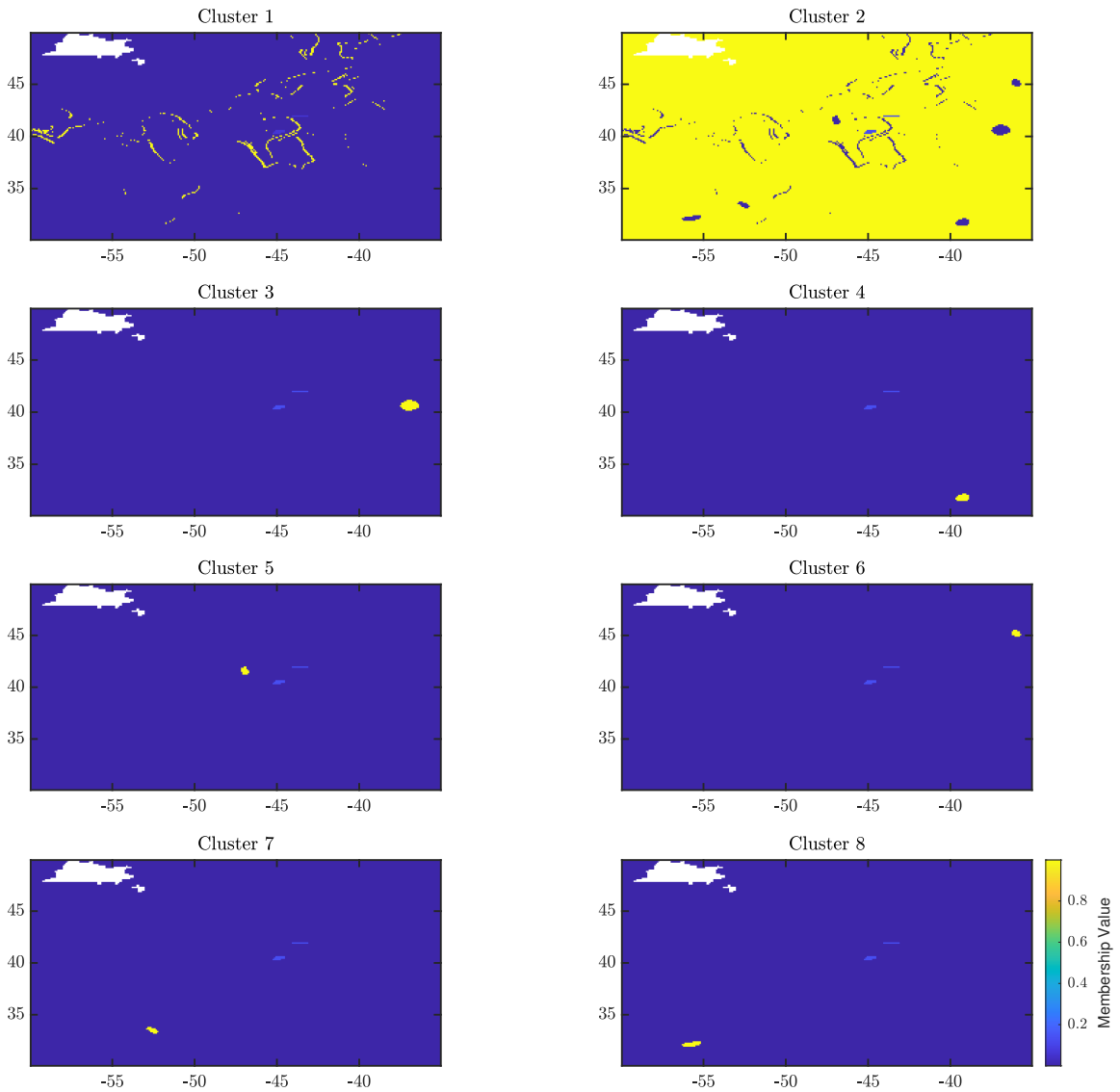
- Grouping similar points together into clusters.
- Key idea: **any** partitioning of the flow is a clustering.
- Fuzzy consensus clustering combines different clusterings of the same data, proposed by Wu et al. (2017).

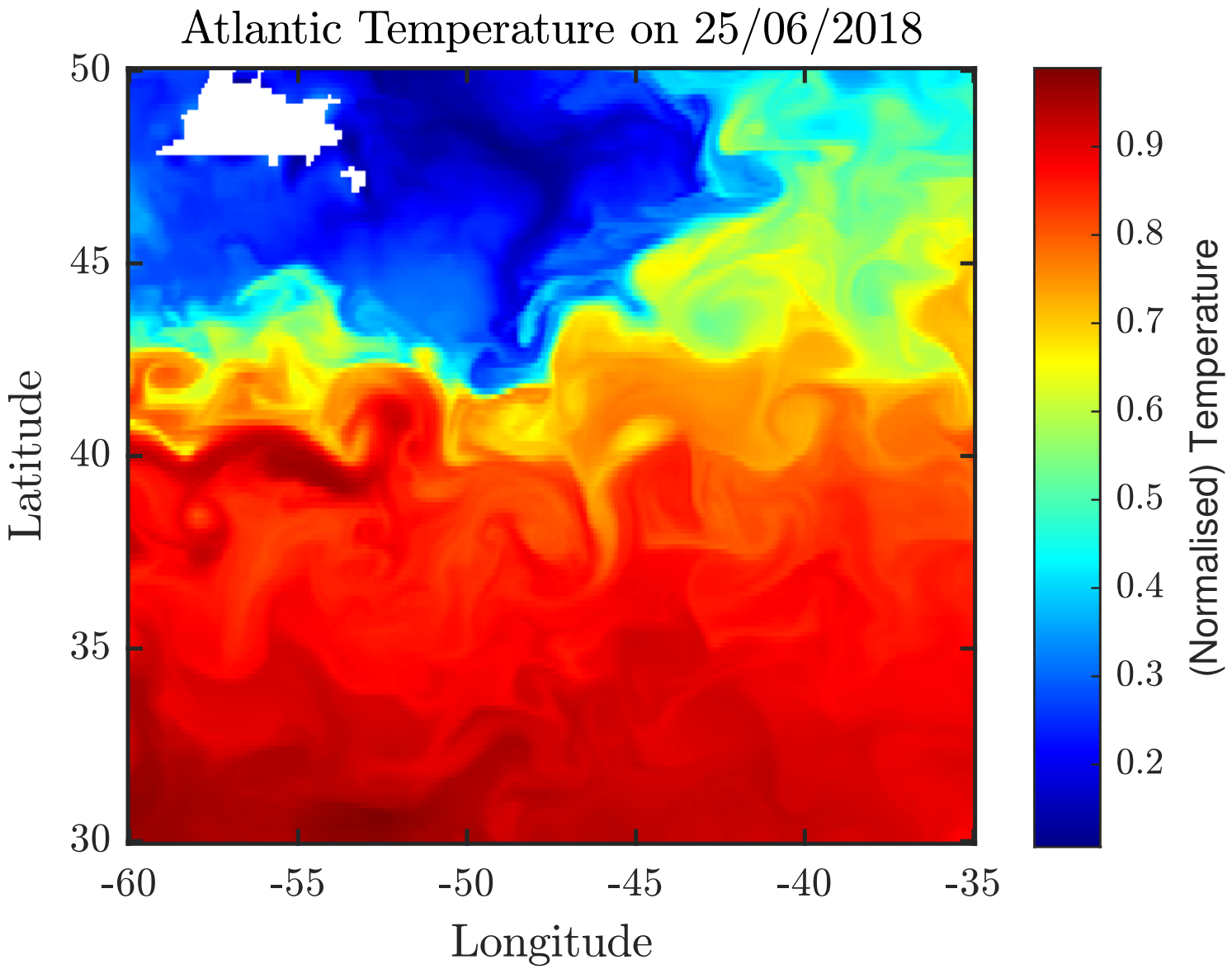

Consensus of FTLE Ridges & LAVD vortices
Cluster 1
Cluster 2
Cluster 3
Cluster 4
Cluster 5
Cluster 6
Cluster 7
Cluster 8
45
40
35
-55
-50
-45
-40
0.8
0.6
0.4
0.2
Membership Value

Temperature Field

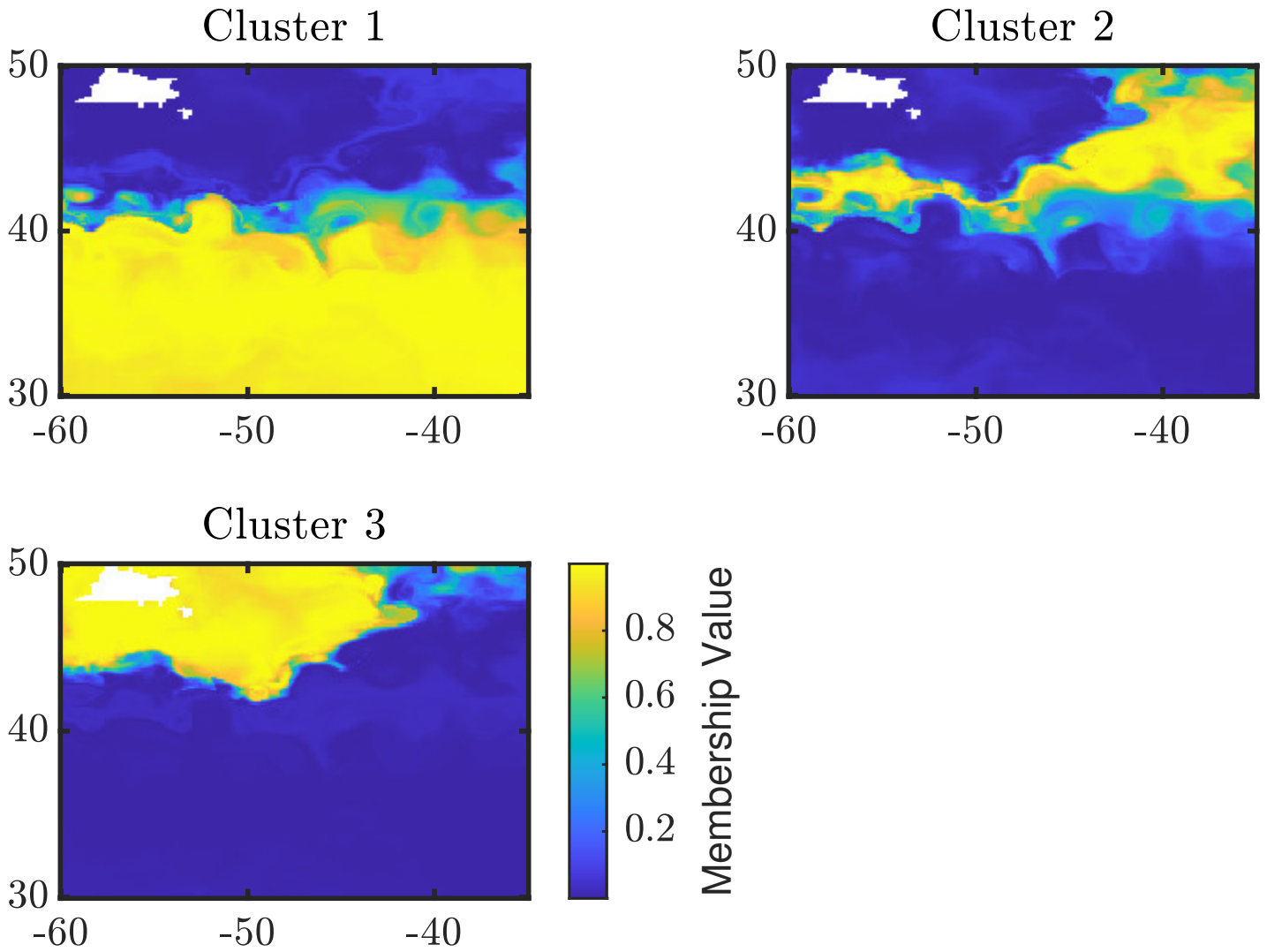
- But coherency can extend beyond just the flow itself...

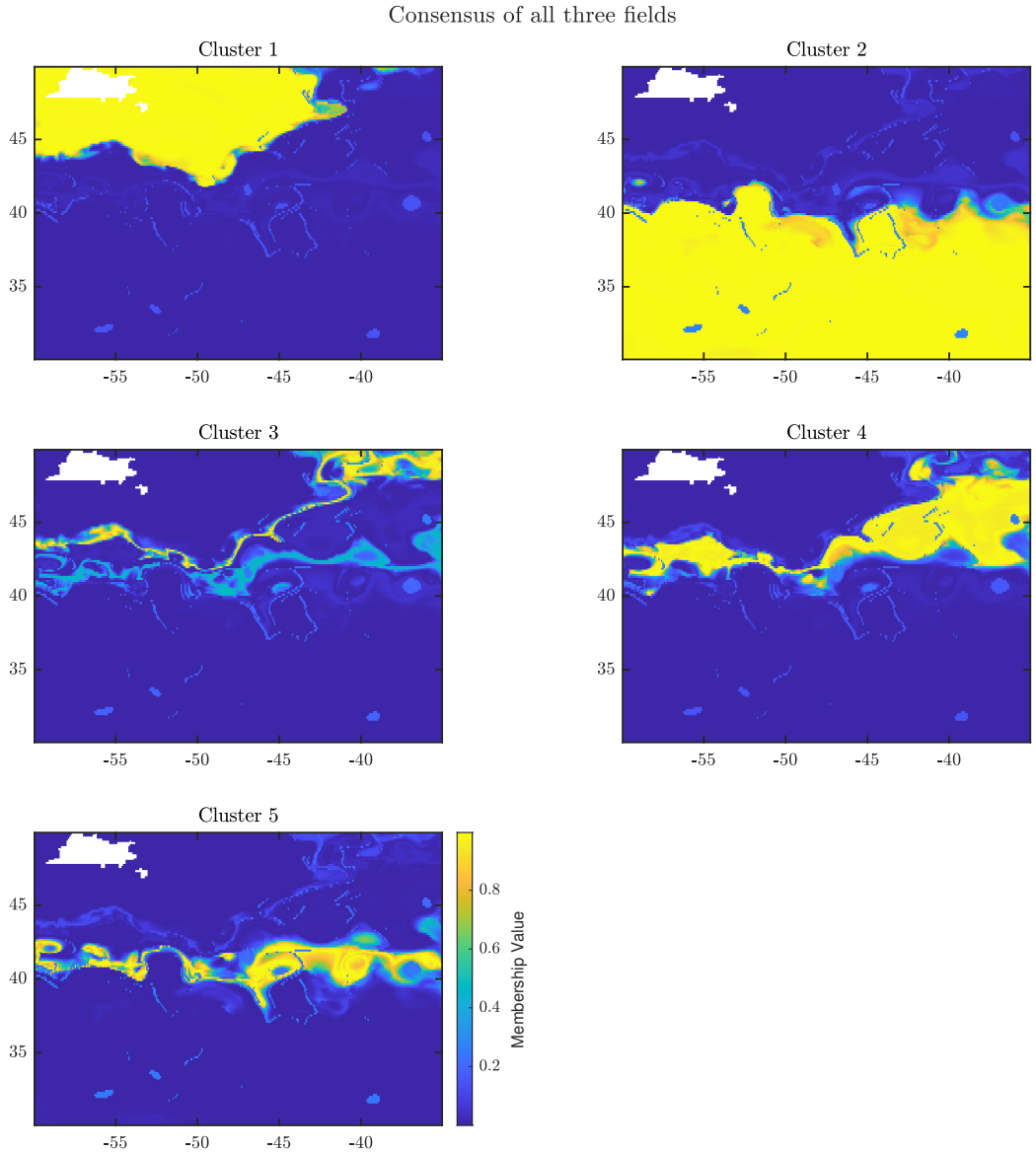
Temperature Field

- But coherency can extend beyond just the flow itself...
- Co-evolving variables such as temperature can reflect LCSs in flow or suggest their own coherency, from Balasuriya et al. (2018).


Atlantic Temperature on 25/06/2018
Latitude
50
45
40
35
30
-60
-55
-50
-45
-40
-35
Longitude
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
(Normalised) Temperature

SST Cluster Membership




Consensus of all three fields
Cluster 1
Cluster 2
Cluster 3
Cluster 4
Cluster 5
45
40
35
-55
-50
-45
-40
0.8
0.6
0.4
0.2
Membership Value

Conclusions

- Combining FTLE & LAVD is not quite right.
- Parameters are tricky.
- Addition of temperature was promising.

References

Balasuriya, S., Ouellette, N. T., and Rypina, I. I. (2018). Generalized Lagrangian coherent structures. *Physica D: Nonlinear Phenomena*, 372:31 – 51.

Haller, G., Hadjighasem, A., Farazmand, M., and Huhn, F. (2016). Defining coherent vortices objectively from the vorticity. *Journal of Fluid Mechanics*, 795:136–173.

Shadden, S. C., Lekien, F., and Marsden, J. E. (2005). Definition and properties of Lagrangian coherent structures from finite-time Lyapunov exponents in two-dimensional aperiodic flows. *Physica D: Nonlinear Phenomena*, 212(3):271 – 304.

Wu, J., Wu, Z., Cao, J., Liu, H., Chen, G., and Zhang, Y. (2017). Fuzzy consensus clustering with applications on big data. *IEEE Transactions on Fuzzy Systems*, 25(6):1430–1445.